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Second-Order Accurate Finite-Difference Scheme for Solving the Problem of Elastic Wave Diffraction by the Anisotropic Gradient Layer

A. A. Anufrieva[1*], E. V. Rung[1], and D. N. Tumakov[1***]**

(Submitted by E. K. Lipachev)

[1] *Institute of Computational Mathematics and Information Technologies, Kazan Federal University, ul. Kremlevskaya 18, Kazan, Tatarstan, 420008 Russia*

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Abstract—The boundary value problem for the Lame equations for the problem of elastic wave diffraction by an anisotropic layer with continuously varying elastic parameters is considered. The original problem is reduced to the boundary value problem for a system of ordinary differential equations of the given form. The finite-difference scheme is obtained by the method of approximation of integral identities. The theorem is proved that the error of approximation of the solution has a second order of accuracy for sufficiently continuous values of the elements of the elasticity tensor. Numerical results confirming theoretical conclusions are given.

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1. INTRODUCTION

Anisotropic materials are the subject of an active research in the geology of the last 50 years [1–4]. One of the applied research directions in this field is seismology [5–7]. Research in this direction makes it possible to deepen the understanding of the structure of the earth's crust—its lithosphere and the asthenosphere [8]. Another broad area of research in the field of anisotropic materials is acoustic metamaterials and phonon crystals [9]. These materials are composite artificial materials of complex structure [10, 11]. The development of studies of this type of materials has made significant progress in the screening process, noise isolation, as well as controlling the amount of energy of transmitted or reflected waves [12].

Anisotropy of materials can be caused by different levels of complexity of the structure of matter [13]. In particular, the transversely isotropic model of matter is characterized by a difference in the moduli of elasticity in mutually perpendicular directions and represents the simplest case of anisotropy. The problems of wave propagation in such media are the most investigated problems. For example, the authors earlier studied the problem of passing a plane elastic wave through layers filled with transversely isotropic materials [14, 15].

For an approximate solution of the problem of the propagation of elastic vibrations, various methods are used [16–18]. Of these, the most known are finite-difference (FD) methods and the finite element method. Each of the methods has its pluses and minuses, their comparison is given, for example, in [19]. We note that the finite element method is laborious, but it allows obtaining high accuracy. However, the simplicity of implementing FD methods [20] gives them wider popularity. For FD methods, different

[*] E-mail: nastya-anufrieva@mail.ru
[**] E-mail: HelenRung@mail.ru
[***] E-mail: dtumakov@kpfu.ru